AVIRAL JEE | NEET | FOUNDATIONS

JEE ADV - PHYSICS

DPP -01

21 JAN 2020

TOPICS : KINEMATICS SOLUTION

1. $$v_x = \frac{dx}{dt} = \frac{d}{dt}(at) = a$$

$$v_y = \frac{dy}{dt} = \frac{d}{dt}(bt^2 + ct) = 2bt + c$$

At $t = 1s, v_y = 2b + c$

The magnitude of velocity at $t = 1s$ is given by

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{a^2 + (2b+c)^2}$$

2. As earth's rotation and revolution are in the same direction, relative to sun

$$v_{day} = v_0 - v_S = v_0 - R\omega$$

and $$v_{night} = v_0 + v_S = v_0 + R\omega$$

So $v_{night} > v_{day}$, *i.e.*, objects on earth move faster in night than in day. However, $v_{night} > v_{day} = 2R\omega \simeq 1$ km/s which is much lesser than $v_0 = 30$ km/s

3. (a) $t_A = \sqrt{\frac{2h}{g}}$ while $t_B = \frac{(u^2 + 2gh)^{1/2} - u}{g}$

(b) $v_A = \sqrt{2gh}$ while $v_B = \sqrt{(u^2 + 2gh)}$

4.

5. $$R_P = R_Q \Rightarrow \frac{(\sqrt{2}u)^2 \sin(2\times 15°)}{g} = \frac{u^2 \sin 2\theta}{g}$$

$$1 = \sin 2\theta$$

$$\theta = 45°$$

6. The co-ordinates of a point on one trajectory relative to other will be

$$x = x_2 - x_1 = (u_2 \cos\theta_2 - u_1 \cos\theta_1)t$$

and

$$y = \left(u_2 \sin\theta_2 t - \frac{1}{2}gt^2\right) - \left(u_1 \sin\theta_1 t - \frac{1}{2}gt^2\right)$$

$$= (u_2 \sin\theta_2 - u_1 \sin\theta_1)t$$

So, $$\frac{y}{x} = \left(\frac{u_2 \sin\theta_2 - u_1 \sin\theta_1}{u_2 \cos\theta_2 - u_1 \cos\theta_1}\right) = \text{constt.} = m$$

or $y = mx$ which is a straight line

7. $$H = \frac{u^2 \sin^2\theta}{2g} \text{ and } R = \frac{u^2 \sin 2\theta}{g}$$

$$\frac{H}{R} = \frac{\sin^2\theta}{2\sin 2\theta} = \frac{\sin^2\theta}{4\sin\theta\cos\theta} = \frac{1}{4}\tan\theta$$

8. $$t_1 = \frac{2u\sin\theta}{g} \text{ and } t_2 = \frac{2u\sin(90° - \theta)}{g} = \frac{2u\cos\theta}{g}$$

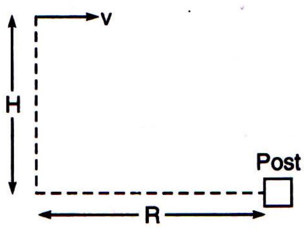
$$\therefore \quad t_1 t_2 = \frac{2u\sin\theta}{g} \times \frac{2u\cos\theta}{g} = \frac{2}{g}\left(\frac{u^2 \sin 2\theta}{g}\right) = \frac{2R}{g}$$

9. Time taken by a body to go up and come down a height h is giving by $t = 2\sqrt{(2h/g)} = \sqrt{(8h/g)}$. So, if y is the height of highest point above higher level, time taken to go up and come down $\Delta T_H = \sqrt{(8y/g)}$ while for lower level $\Delta T_L = \sqrt{[8(y+h)/g]}$.

$$\therefore \quad \Delta T_L^2 - \Delta T_H^2 = \frac{8(y+H)}{g} - \frac{8y}{g} = \frac{8H}{g}$$

or $$g = \frac{8H}{(\Delta T_L^2 - \Delta T_H^2)}$$

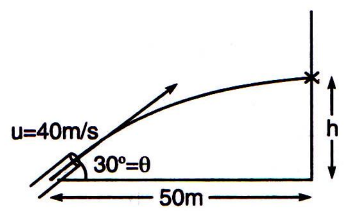
10.



$$H = \frac{1}{2}gt^2$$

$$R = v \times T$$

11.



$$h = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$$

12. $$R_{max} = \frac{u^2}{g}$$

$$H_{max} = \frac{u^2}{2g}$$

13.

14. Let the initial position vector is $x'\hat{\mathbf{i}} + y'\hat{\mathbf{j}} + z'\hat{\mathbf{k}} = \vec{\mathbf{r}}_A$.

Now, $a_x = \frac{12}{3}$, $a_y = -\frac{3}{3}$, $a_z = \frac{21-10}{3}$

$$\vec{\mathbf{r}}_B = 15\hat{\mathbf{i}} + 7\hat{\mathbf{j}} - 6\hat{\mathbf{k}}$$

Now, $15 - x' = u_{x'} \times 2 + \frac{1}{2} \times 4 \times (2)^2$

$$7 - y' = u_{y'} \times 2 + \frac{1}{2}(-1) \times (2)^2$$

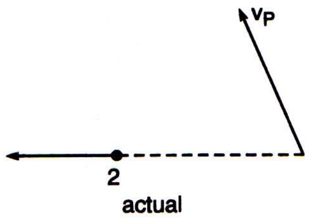
$$-6 - z' = u_{z'} \times 2 + \frac{1}{2} \times \frac{11}{3} \times (2)^2$$

$$\vec{\mathbf{v}}_B = 12\hat{\mathbf{i}} + \hat{\mathbf{j}} - 4\hat{\mathbf{k}}$$

$$\vec{\mathbf{v}}_B = v_x\hat{\mathbf{i}} + v_y\hat{\mathbf{j}} + v_z\hat{\mathbf{k}}$$

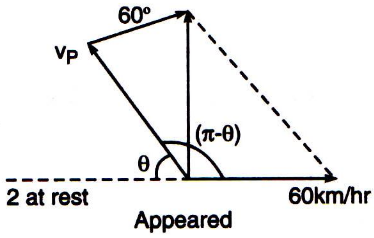
$$\frac{v_x - u_{x'}}{a_x} = t, \quad \frac{v_y - u_{y'}}{a_y} = t, \quad \frac{v_z - u_{z'}}{a_z} = t$$

14. For observer 2.



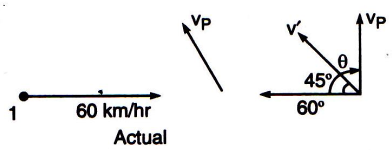
$$v'^2 = v_p^2 + (60)^2$$

$$\tan 90^\circ = \frac{v_P \sin(\pi - \theta)}{60 + v_P \cos(\pi - \theta)}$$



$v_P \cos\theta = 60^\circ$ ... (ii)

For observer 1.



$$\tan 45^\circ = \frac{v_P \sin\theta}{60^\circ + v_P \cos\theta}$$

$$60 + v_P \cos\theta = v_P \sin\theta$$

$v_P \sin\theta = 120^\circ$ ... (iii)

$$\tan\theta = 2$$

15. Position vector of ship after 't' hours from 12 noon is,

$\vec{\mathbf{r}}_s = \vec{\mathbf{r}}_{os} + \vec{\mathbf{v}}_s t = (3\hat{\mathbf{i}} - \hat{\mathbf{j}}) + (3\hat{\mathbf{i}} + 4\hat{\mathbf{j}})t$ ... (i)

Position vector of motor boat at that instant

$\vec{\mathbf{r}}_B = (-6\hat{\mathbf{i}} - 2\hat{\mathbf{j}}) + (a\hat{\mathbf{i}} + b\hat{\mathbf{j}})(1+t)$ ... (ii)

where $a\hat{\mathbf{i}} + b\hat{\mathbf{j}} = \vec{\mathbf{v}}_B$ (velocity of motor boat)

Given $a^2 + b^2 = 53$ ... (iii)

when they intercept, $\vec{\mathbf{r}}_s = \vec{\mathbf{r}}_B$

Equate coefficients of $\hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$ and use equation (iii)

we get $b = 2$ and $\frac{34}{5}$; with $b = \frac{34}{5}$ time 't' comes out to be negative which is not possible so $b = 2$,

from this $a = 7$ and $t = \frac{1}{2}$.

Position vector of point of interception is $\frac{a}{2}\hat{\mathbf{i}} + \hat{\mathbf{j}}$.

16. $|\vec{V}_{M/R}| = 4.0$ km/hr,

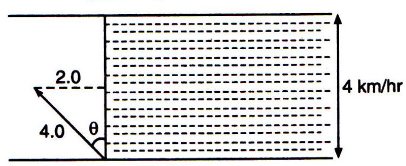
$|\vec{V}_{R/G}| = 2.0$ km/hr

$$\sin\theta = \frac{2}{4} = \frac{1}{2}$$

$$\theta = 30^\circ$$

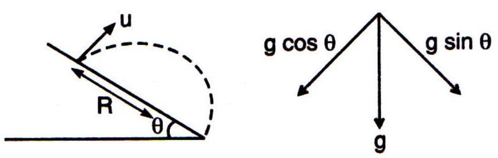
$$t = \frac{4.0}{4\cos 30^\circ} = \frac{2}{\sqrt{3}}\text{ hr}$$

$$d_{\text{drift}} = V_R \times t$$



In order to cross the river in minimum time he should go perpendicular to river flow.

17.



$$R = \frac{1}{2} g \sin\theta \,(T)^2$$

$$T = \frac{2u}{g\cos\theta}$$

18.

$$T = \frac{2u}{g} = \frac{2\times 20}{10} = 4\,\text{sec}$$

Distance travelled by the cannon.

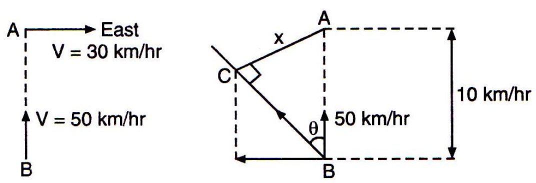
$$s = 30\times 4\times \frac{5}{18}\,\text{m}$$

$$s = 120\times\frac{5}{18}$$

$$s = \frac{100}{3}\,\text{m}$$

19.

$$\frac{x}{10} = \sin\theta$$



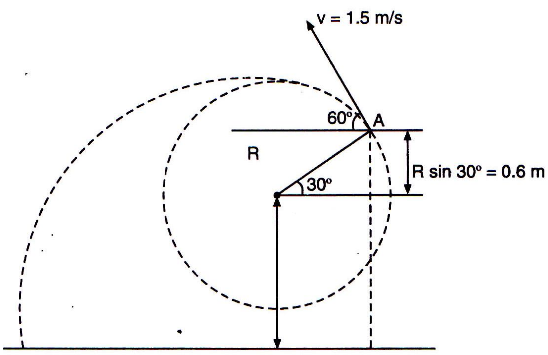
$x = 10\sin\theta$ Closest approach

$$t = \frac{BC}{\sqrt{(30)^2 + (50)^2}}$$

20. (a) $h = 1.5 + 0.6 = v\sin 60^\circ t - \frac{1}{2}gt^2$

$R_1 = v\cos 60^\circ t$

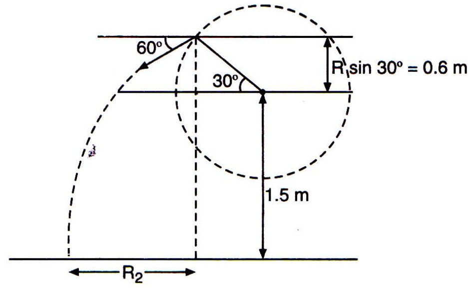
Solving $R_1 = 600$ m



(b) $h = 1.5 + 0.6 = -v\sin 60^\circ t - \frac{1}{2}gt^2$

$R_1 = v\cos 60^\circ t$

Solving $R_2 = 0.402$ m



(c) $a = \frac{v^2}{R} = \frac{(1.5)^2}{1.2} = 1.87\ \text{m/s}^2$ towards centre

(d) $a = g = 9.8\ \text{m/s}^2$ down

21. Velocity of 1st particle at any instant 't'

$$\vec{v}_1 = u_1\cos\theta_1\hat{i} + (u_2\sin\theta_2 - gt)\hat{j} \quad ...(i)$$

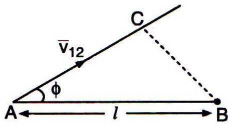
Velocity of 2nd particle at the same instant 't'

$$\vec{v}_2 = u_2\cos\theta_2(-\hat{i}) + (u_2\sin\theta_2 - gt)\hat{j} \quad ...(ii)$$

∴ Velocity of 1st particle w.r.t. 2nd particle

$$\vec{v}_{12} = \vec{v}_1 - \vec{v}_2 = (u_1\cos\theta_1 + u_2\cos\theta_2)\hat{i} + (u_1\sin\theta_1 - u_2\sin\theta_2)\hat{j} \quad ...(iii)$$

$$\tan\phi = \frac{u_1\sin\theta_1 - u_2\sin\theta_2}{u_1\cos\theta_1 + u_2\cos\theta_2} \quad ...(iv)$$



(a) Separation when they are closest is $BC = l\sin\phi$

(b) $t = \frac{AC}{|\vec{v}_{12}|} = \frac{l\cos\phi}{|\vec{v}_{12}|}$;

$$|\vec{v}_{12}| = \sqrt{(u_1\cos\theta_1 + u_2\cos\theta_2)^2 + (u_1\sin\theta_1 - u_2\sin\theta_2)^2}$$

(c) If they collide then BC (minimum separation) = 0

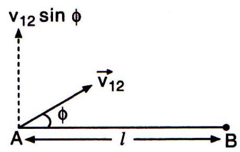
i.e., $\phi = 0$

∴ $\tan\phi = 0$

⇒ $u_1\sin\theta_1 = u_2\sin\theta_2$ from equation (iv)

(d) Velocity of approach

$= u_1\cos\theta_1 + u_2\cos\theta_2$



(e) $\omega_{12} = \omega_{AB} = \frac{v_{12}\sin\phi}{l}$

$$\omega_{12} = \frac{u_1\sin\theta_1 - u_2\sin\theta_2}{l}$$